



# 3D Analysis of UAVSAR Repeat-pass Interferometry Data

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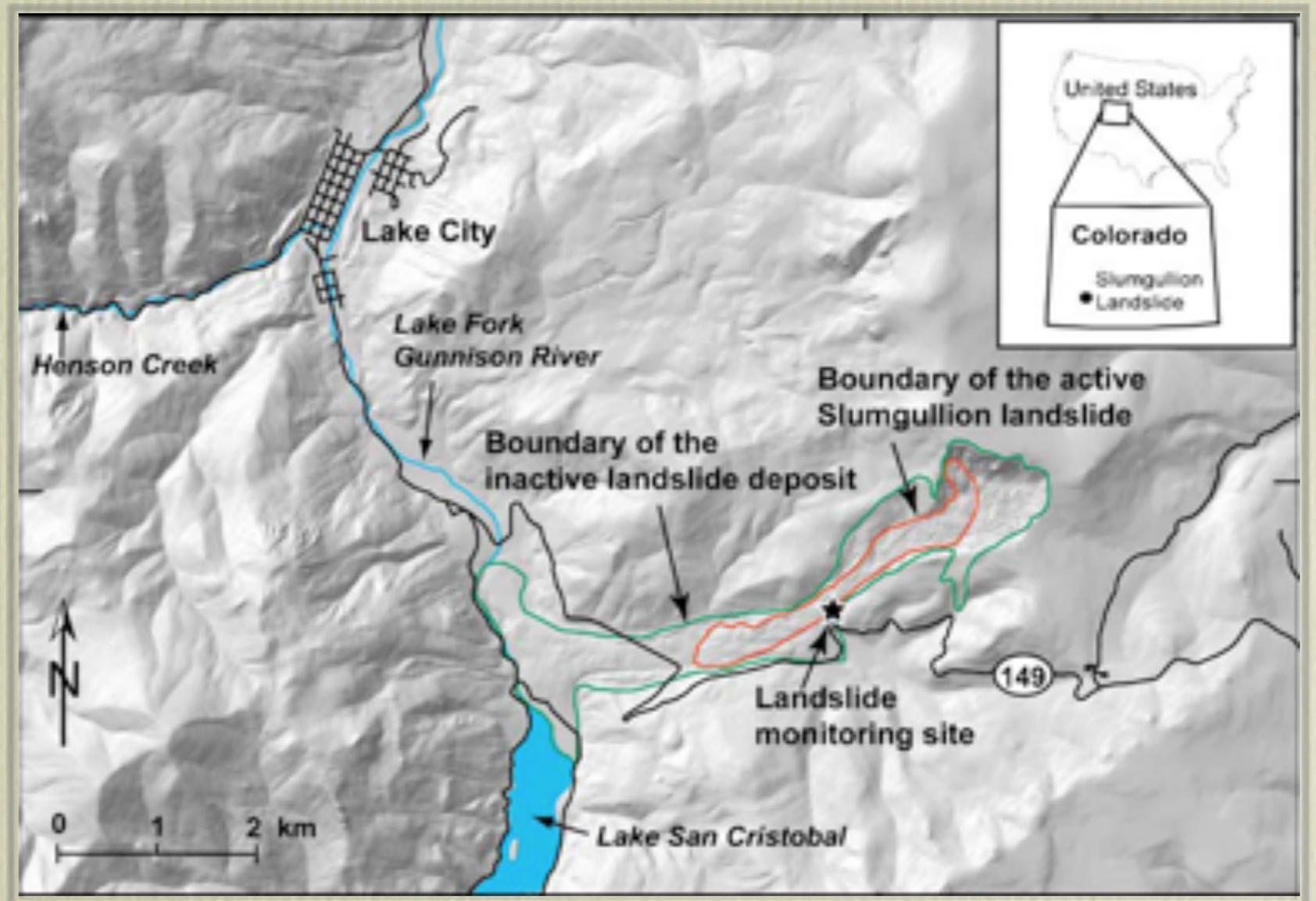
## Three UAVSAR projects

- Slumgullion, SW Colorado (2011–): rapidly moving landslide—3D surface motion measurement (also Delbridge et al. talk Thurs.)
- San Francisco Bay area (2008–): Hayward Fault, Berkeley Hills landslides, other faults and deformation
- Salton Trough, Mexico (2012–): Postseismic deformation related to 2010 M7.2 earthquake, Cerro Prieto geothermal field

# The Slumgullion Natural Laboratory

The active Slumgullion Landslide:

- ✿ Velocity: 1-2 cm/day
- ✿ Average Slope: 8 degrees
- ✿ Length: 3.9 km
- ✿ Width: ~300 m
- ✿ Depth: ~14 m
- ✿ Volume:  $20 \times 10^6 \text{ m}^3$



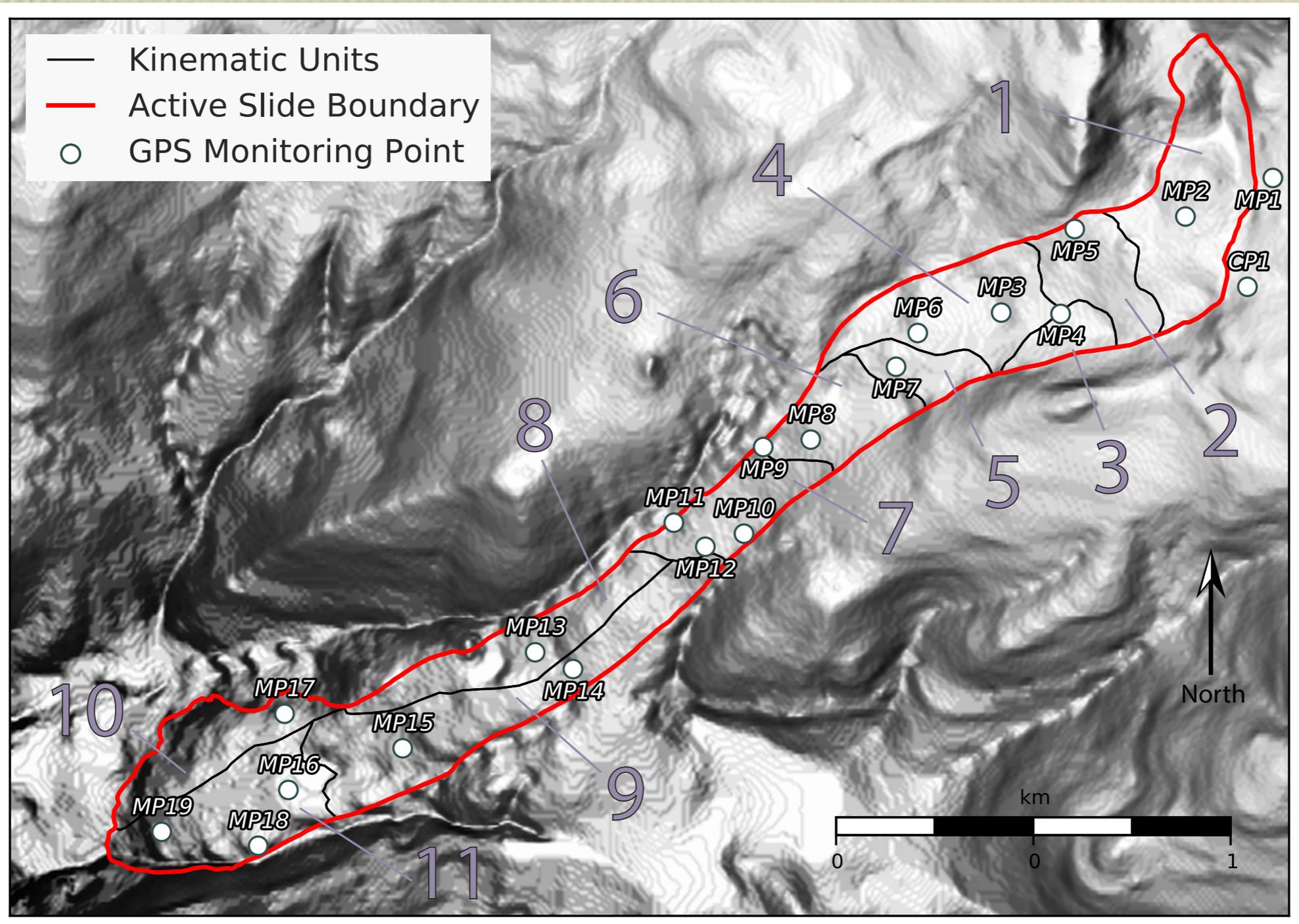
**Image From Schulz et al 2009**

# The Slumgullion Natural Laboratory

- ✿ The rapid deformation rate allows us to observe the deformation on the timescale of days
- ✿ The large spatial extent of the slide allows to explore complex interactions between distinct kinematic units
- ✿ The slide's continuous motion allows us to observe its response to environmental forcing

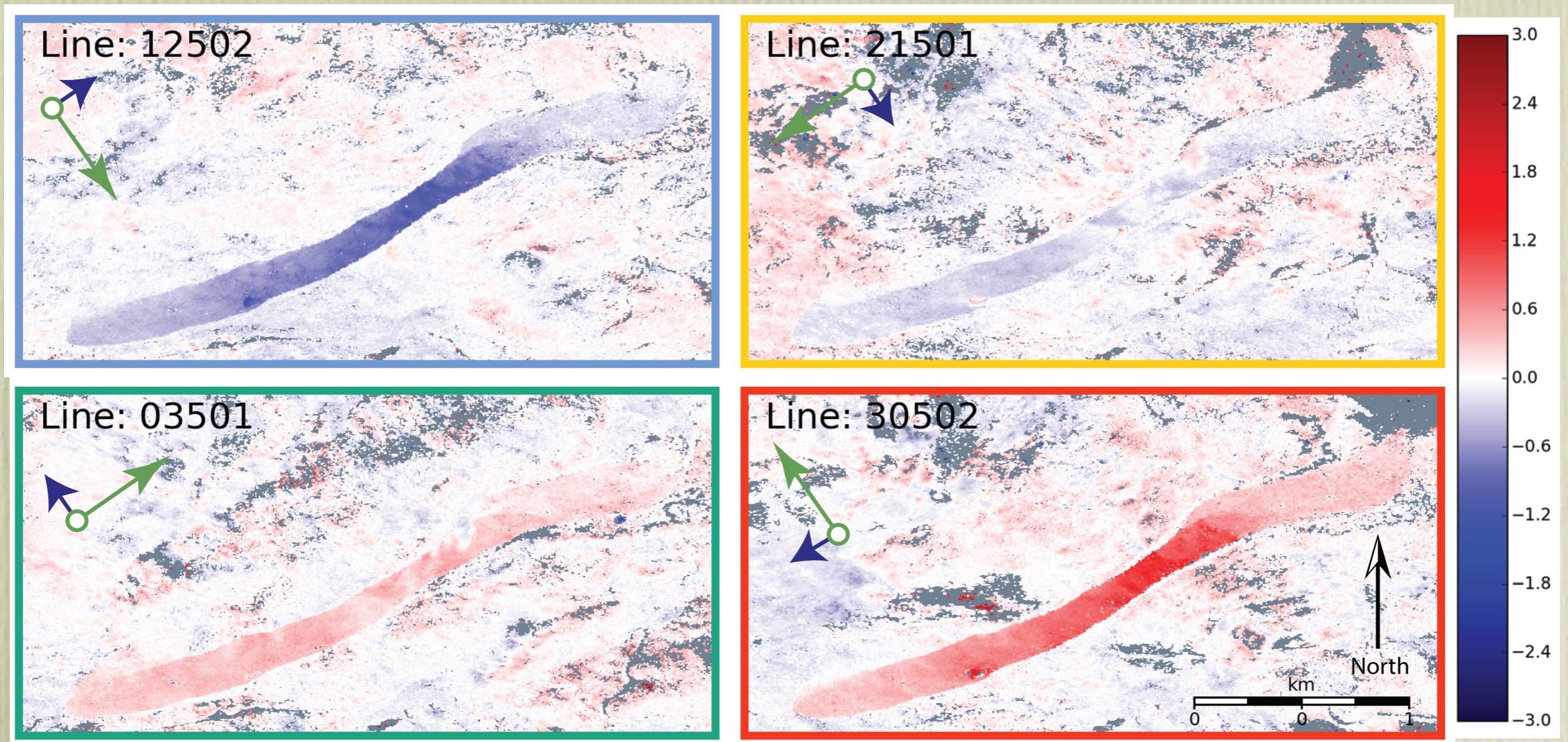


# Distinct Kinematic Units



**Units defined by W. Schulz (2012) based on field mapping**

# UAVSAR 7-Day Interferograms



**B. DELBRIDGE ANALYSIS**

# THREE DIMENSIONAL VECTOR INVERSION

To obtain the full vector deformation of the Landslide motion we combine the deformation from the four LOS observations.

•The deformation vector given in terms of the basis vectors North, East, and Vertical is given by:

$$\vec{d} = \sum_{i=1}^3 d_i \hat{e}_i = \sum_{i=1}^3 \langle \vec{d}, \hat{e}_i \rangle \hat{e}_i$$

For each LOS interferogram we observe the true deformation vector projected on the LOS direction, and from the above EQ:

$$o_j = \langle \vec{d}, \hat{l}_j \rangle = \sum_{i=1}^3 d_i \langle \hat{e}_i, \hat{l}_j \rangle$$

# THREE DIMENSIONAL VECTOR INVERSION

THIS PROBLEM HAS NOW BEEN FORMULATED AS A CLASSICAL LEAST SQUARES PROBLEM FOR EACH PIXEL:

$$\vec{o} = A\vec{d}$$

WHERE  $\mathbf{O}$  (4X1) IS THE VECTOR OF LOS OBSERVATIONS, AND THE COMPONENTS OF  $\mathbf{A}$  (4X3) ARE SIMPLY THE DIFFERENT LOS VECTORS(ROWS) PROJECTED ONTO THE CORRESPONDING BASIS VECTOR(COLUMNS). THUS THE DESIRED DISPLACEMENT VECTOR  $\mathbf{D}$  IS GIVEN BY:

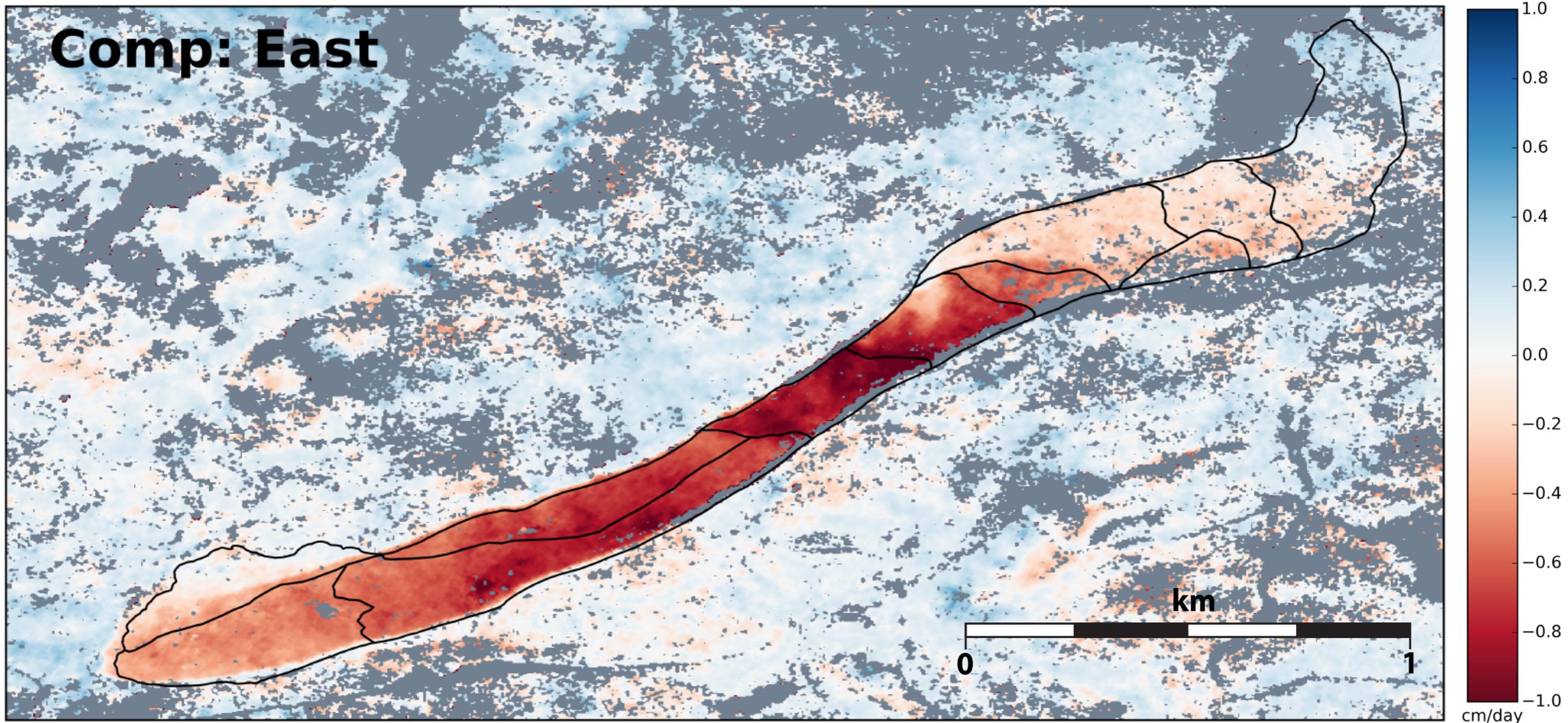
$$\vec{d} = \left( A^t Q^{-1} A \right)^{-1} A^t Q^{-1} \vec{o}$$

$\mathbf{Q}$  IS THE ESTIMATED COVARIANCE MATRIX AND IS CALCULATED FROM THE PIXEL CORRELATION USING THE CRAMER-RAO BOUNDS DERIVED IN SEYMOR 1994.

$$Q = \frac{\lambda}{4\pi} \sqrt{\frac{1 - \gamma^2}{2N_L \gamma^2}} \mathbf{I}$$

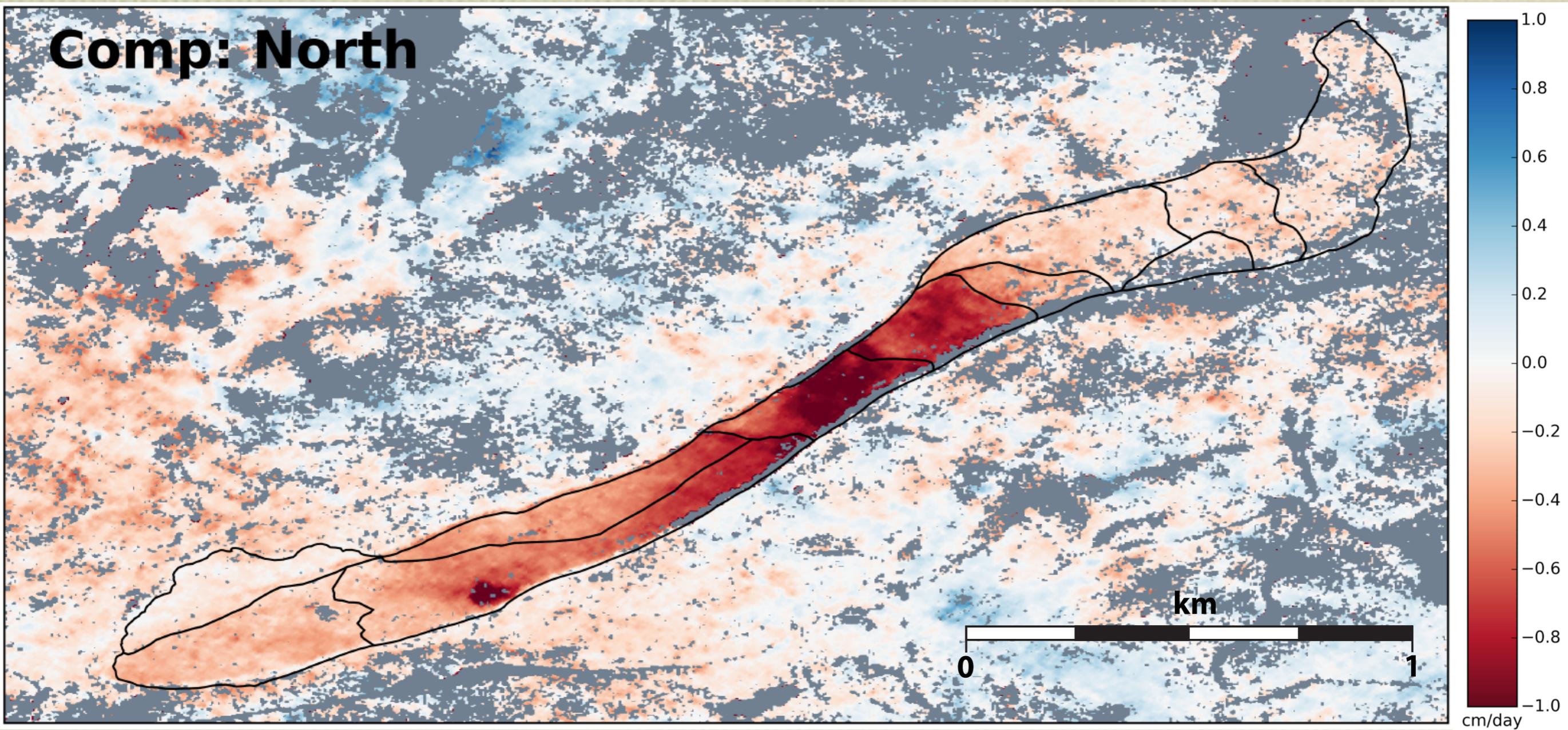
# Results of the Three Dimensional Vector Inversion

**Comp: East**



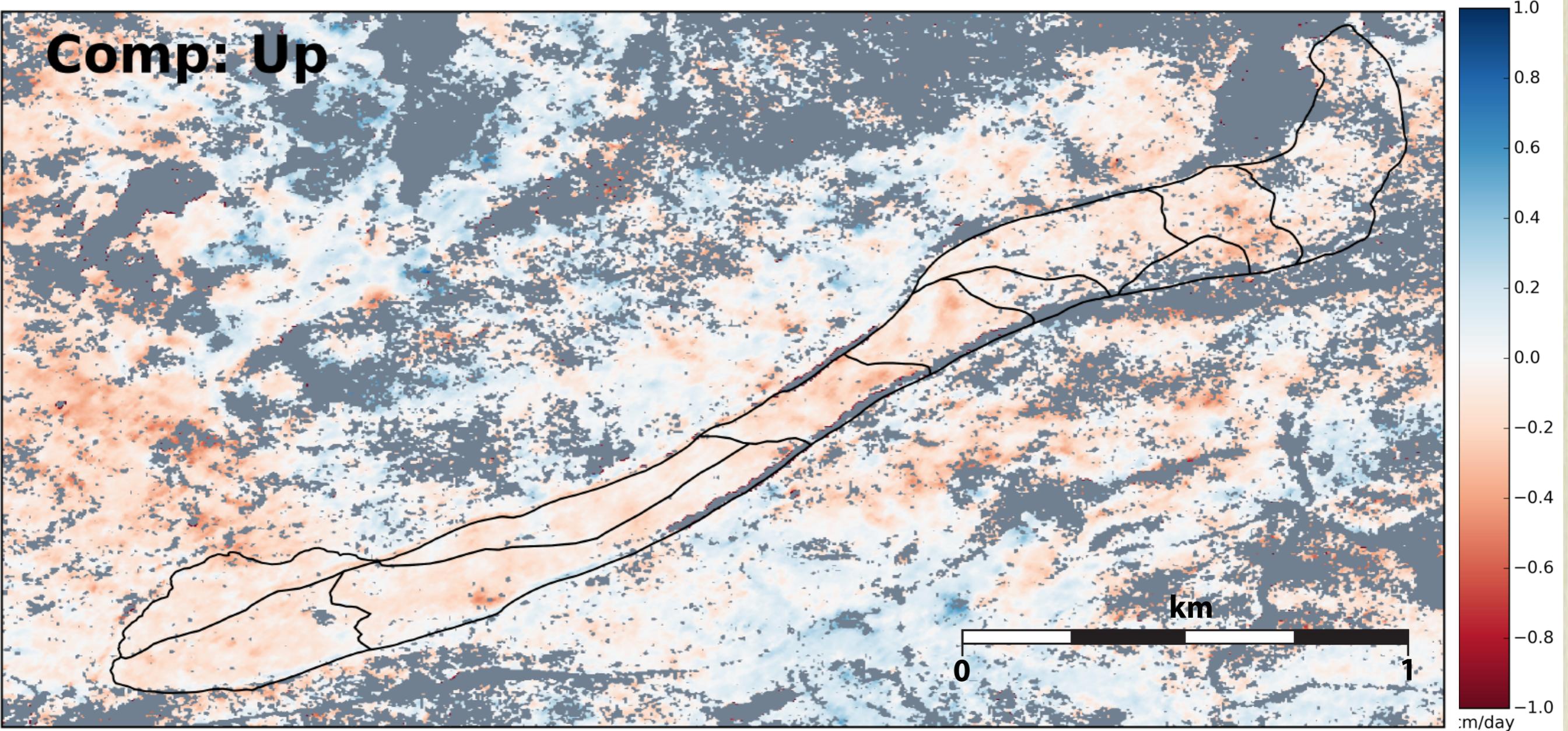
# Results of the Three Dimensional Vector Inversion

**Comp: North**

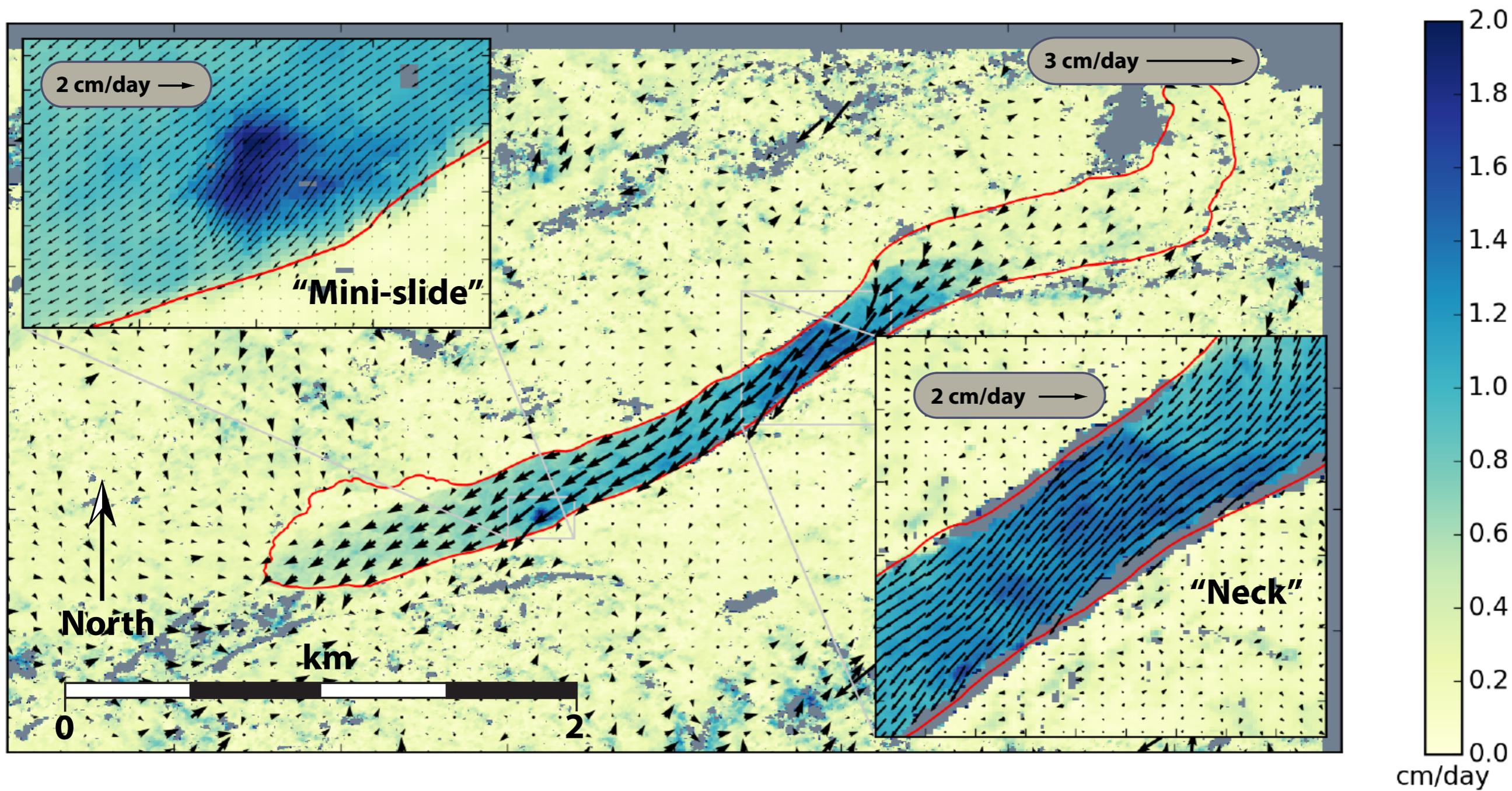


# Results of the Three Dimensional Vector Inversion

**Comp: Up**

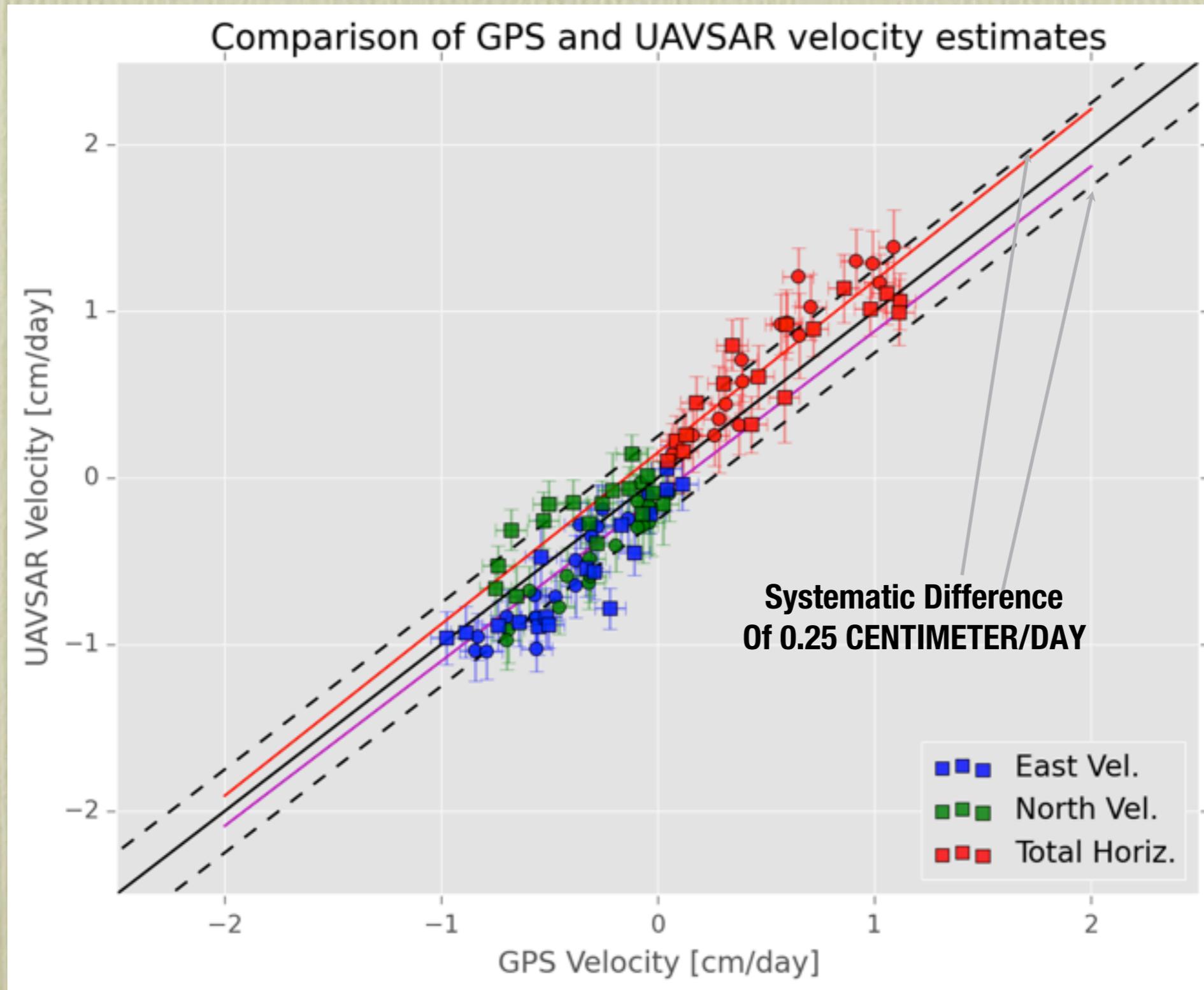


# Horizontal Motion



# Horizontal Motion

Comparison with concurrent campaign GPS data



# 3D Conclusions

- In order to overcome spatial and temporal limitations of traditional spaced-based InSAR and ground-based displacement measurements we present a method for the characterization of 3D surface deformation using the unique capabilities of the NASA/JPL UAVSAR airborne repeat-pass interferometry system.
- A comparison with GPS measurements validates this method and shows that it provides reliable and accurate 3D surface measurements.
- The data acquisition and processing scheme presented here can be used to measure 3-D surface deformation of any kind with applications to hydrology, seismology, and volcanology.

- The differential interferometric phase measurement is given by

$$\phi = \frac{4\pi}{\lambda} \left[ -\langle \vec{b}, \hat{\ell} \rangle + \langle \vec{d}, \hat{\ell} \rangle + \Delta\rho_{atm} \right] + \phi_{noise}$$

↓

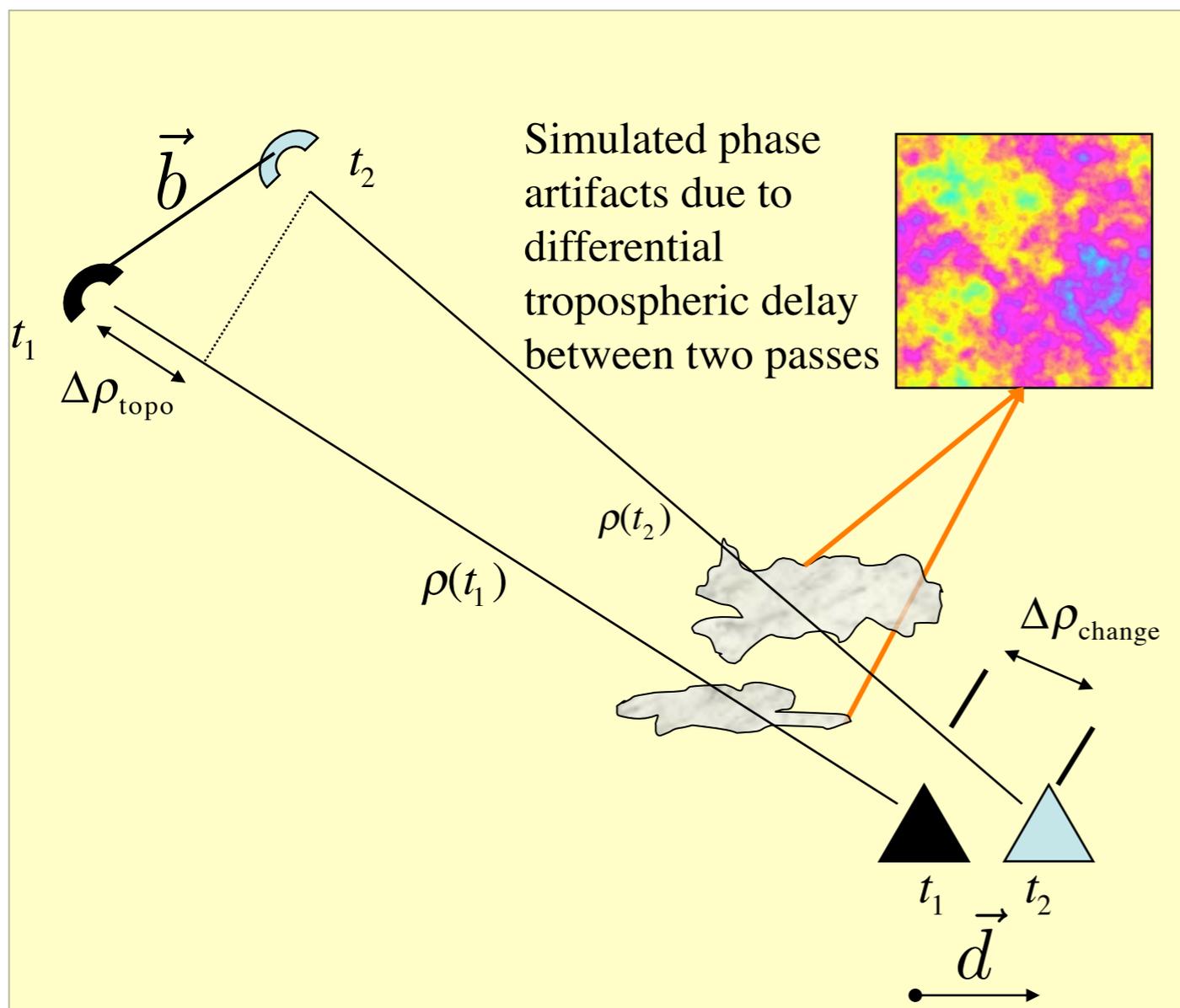
**Topography  
Term**

↓

**Change  
Term**

↓

**Atmosphere  
Term**



- Topography term is assumed known and removed for remainder of discussion
- Measurement only of surface displacement along line-of-sight that can not be distinguished from tropospheric path delay
- Tropospheric path delays cause artifacts in repeat-pass interferometric synthetic aperture radar (InSAR) measurements of surface displacement
  - Rapidly varying tropospheric delays (both spatially and temporally) are most problematic
  - Such variations are primarily due to changes in water vapor content along signal propagation path

- To obtain full vector deformation measurements multiple measurements from different line-of-sights are needed that when combined give the deformation vector in the desired reference frame.
- The vector displacement in terms of a specified set of basis vectors is given by

$$\vec{d} = \sum_{i=1}^3 d_i \hat{e}_i = \sum_{i=1}^3 \langle \vec{d}, \hat{e}_i \rangle \hat{e}_i$$

- Suppose we have N deformation observations,  $o_j = \langle \vec{d}, \hat{\ell}_j \rangle$   $j=1, N$  along line-of-sights  $\hat{\ell}_j$  then from the above equation

$$\langle \vec{d}, \hat{\ell}_j \rangle = \sum_{i=1}^3 d_i \langle \hat{\ell}_j, \hat{e}_i \rangle$$

and hence the sensitivity of the  $i^{\text{th}}$  component of the deformations is

$$\frac{\partial o_j}{\partial d_i} = \langle \hat{\ell}_j, \hat{e}_i \rangle$$

- The set of N in observations can be written in matrix form as

$$\vec{o} = \begin{bmatrix} \langle \vec{d}, \hat{\ell}_1 \rangle \\ \vdots \\ \langle \vec{d}, \hat{\ell}_N \rangle \end{bmatrix}_{N \times 1} = \begin{bmatrix} \langle \hat{\ell}_1, \hat{e}_1 \rangle & \langle \hat{\ell}_1, \hat{e}_2 \rangle & \langle \hat{\ell}_1, \hat{e}_3 \rangle \\ \vdots & \vdots & \vdots \\ \langle \hat{\ell}_N, \hat{e}_1 \rangle & \langle \hat{\ell}_N, \hat{e}_2 \rangle & \langle \hat{\ell}_N, \hat{e}_3 \rangle \end{bmatrix}_{N \times 3} \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}_{3 \times 1}$$

$$\vec{o} = A \vec{d}$$

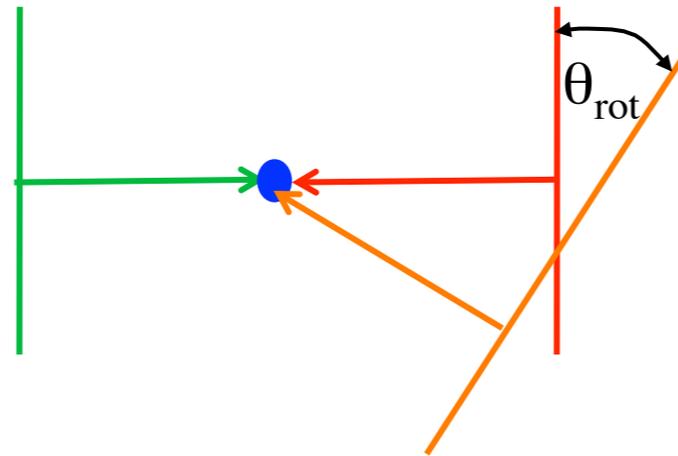
which is classical least squares problem with solution

$$\vec{d} = (A^t Q^{-1} A)^{-1} A^t Q^{-1} \vec{o}$$

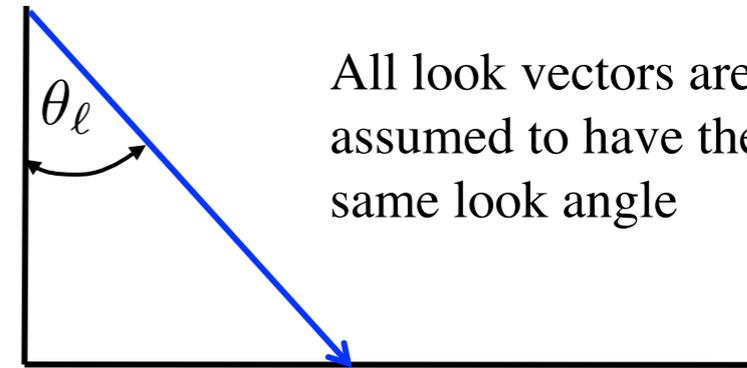
$$\vec{d} = \begin{bmatrix} \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{\ell}_j \rangle^2 & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{\ell}_j \rangle \langle \hat{e}_2, \hat{\ell}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{\ell}_j \rangle \langle \hat{e}_3, \hat{\ell}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{\ell}_j \rangle \langle \hat{e}_1, \hat{\ell}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{\ell}_j \rangle^2 & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{\ell}_j \rangle \langle \hat{e}_3, \hat{\ell}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{\ell}_j \rangle \langle \hat{e}_1, \hat{\ell}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{\ell}_j \rangle \langle \hat{e}_2, \hat{\ell}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{\ell}_j \rangle^2 \end{bmatrix}^{-1} \begin{bmatrix} \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{\ell}_j \rangle \langle \vec{d}, \hat{\ell}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{\ell}_j \rangle \langle \vec{d}, \hat{\ell}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{\ell}_j \rangle \langle \vec{d}, \hat{\ell}_j \rangle \end{bmatrix}$$

- The vectors deformation accuracy is then

$$\sigma_{\vec{d}} = \text{diag} \begin{bmatrix} \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{l}_j \rangle^2 & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{l}_j \rangle \langle \hat{e}_2, \hat{l}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_1, \hat{l}_j \rangle \langle \hat{e}_3, \hat{l}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{l}_j \rangle \langle \hat{e}_1, \hat{l}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{l}_j \rangle^2 & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_2, \hat{l}_j \rangle \langle \hat{e}_3, \hat{l}_j \rangle \\ \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{l}_j \rangle \langle \hat{e}_1, \hat{l}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{l}_j \rangle \langle \hat{e}_2, \hat{l}_j \rangle & \sum_{j=1}^N \frac{1}{\sigma_j^2} \langle \hat{e}_3, \hat{l}_j \rangle^2 \end{bmatrix}^{-1}$$



Downward View

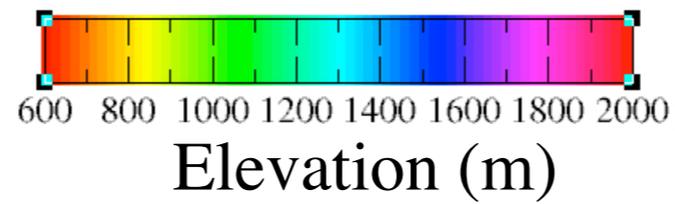
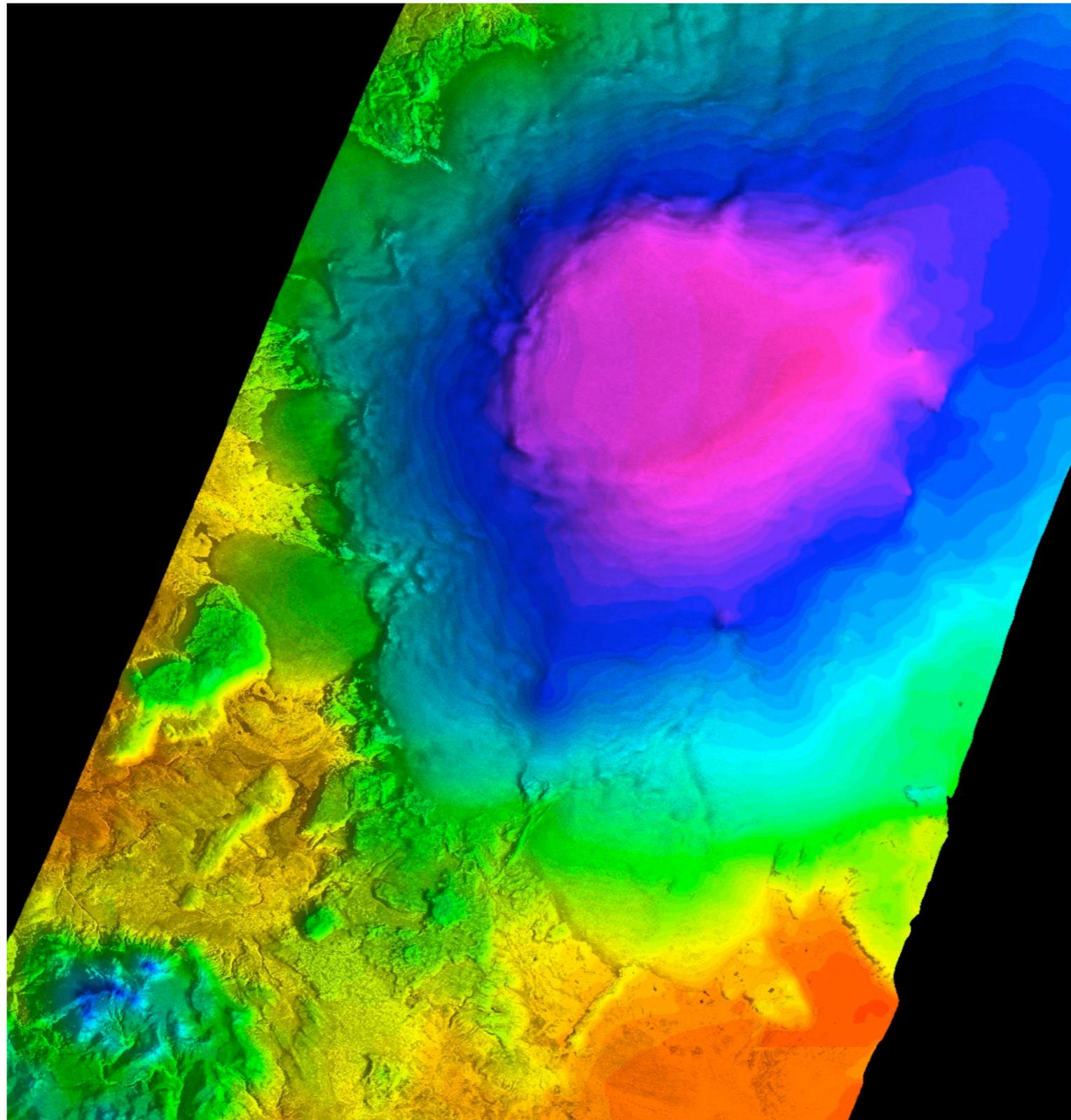


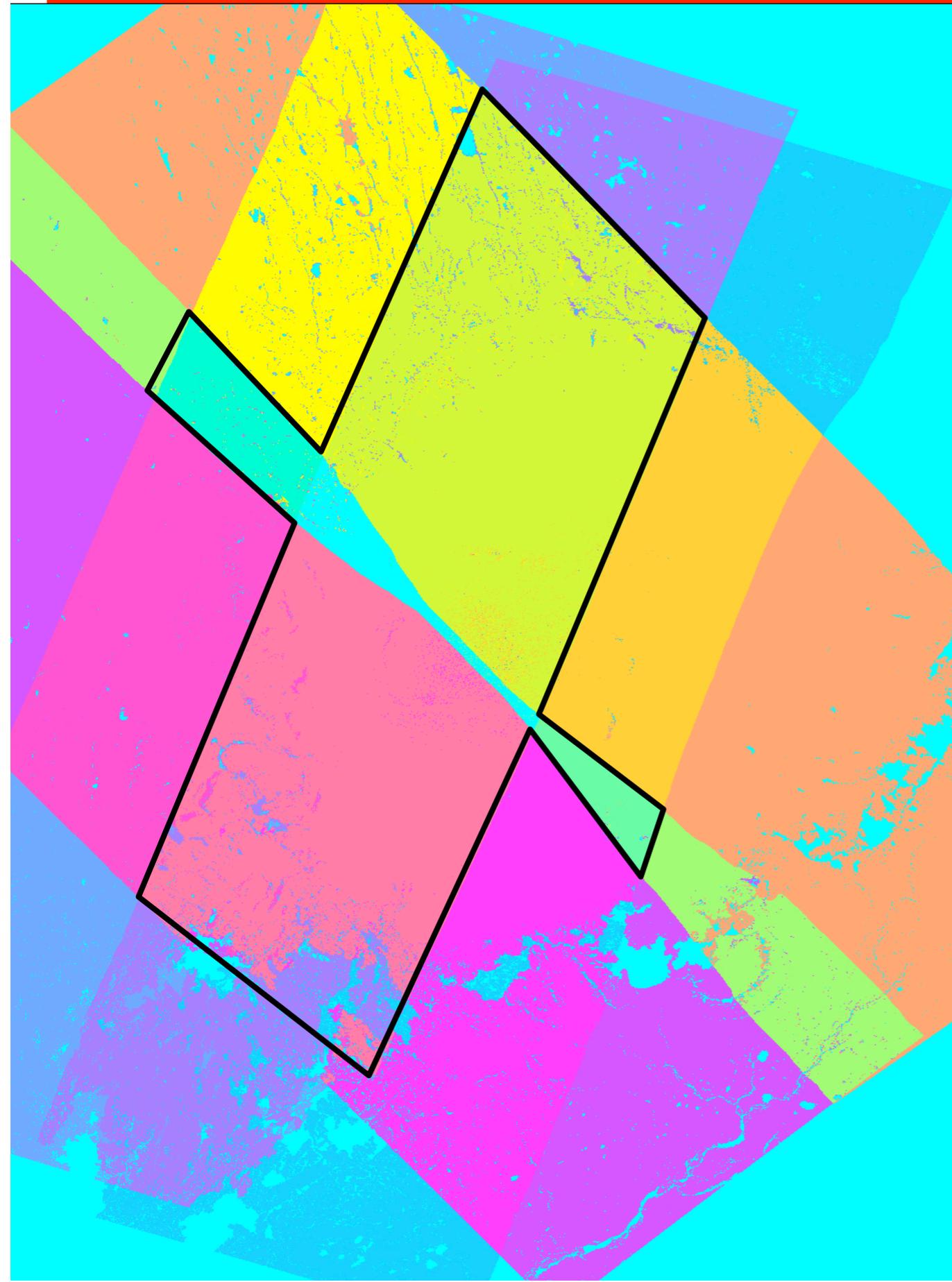
All look vectors are assumed to have the same look angle

Side View

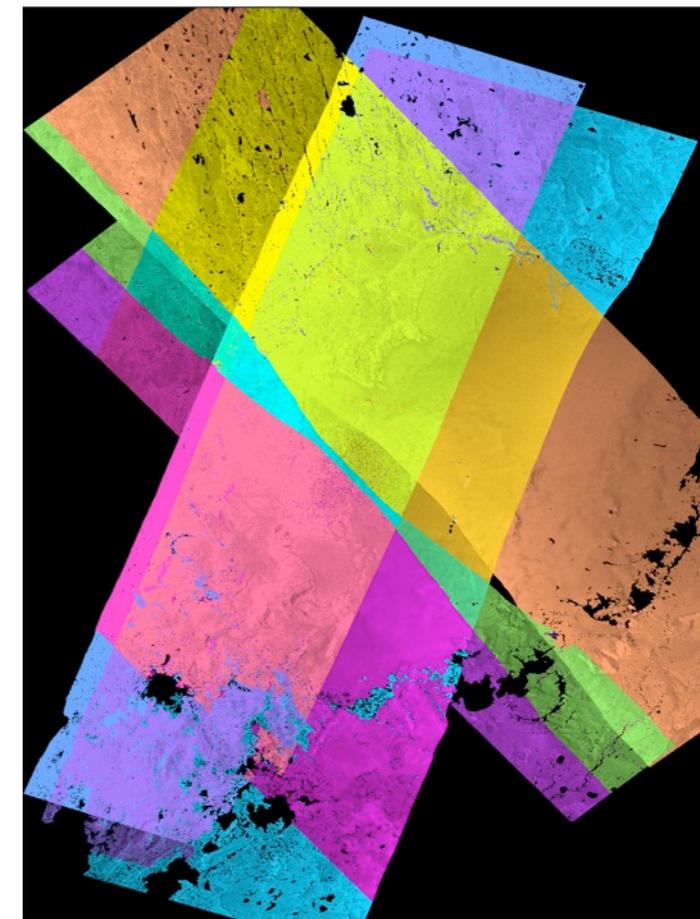
- For the geometry above, the matrix has the form

$$\sigma_{\vec{d}} = \left( \frac{\lambda}{4\pi} \right)^2 \frac{1}{2N_L} \frac{1 - \gamma^2}{\gamma^2} \begin{bmatrix} \frac{1}{4} \frac{7 + \cos(2\theta_{rot})}{\sin^2(\theta_\ell) \sin^2(\theta_{rot})} & \frac{1}{2} \frac{1}{\tan(\theta_{rot}) \tan^2(\theta_\ell)} & \frac{-1}{\sin(2\theta_\ell) \sin(\theta_{rot})} \\ \frac{1}{2} \frac{1}{\tan(\theta_{rot}) \tan^2(\theta_\ell)} & \frac{1}{2 \sin^2(\theta_\ell)} & 0 \\ \frac{-1}{\sin(2\theta_\ell) \sin(\theta_{rot})} & 0 & \frac{1}{2 \cos^2(\theta_\ell)} \end{bmatrix}$$



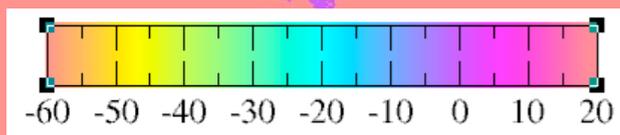
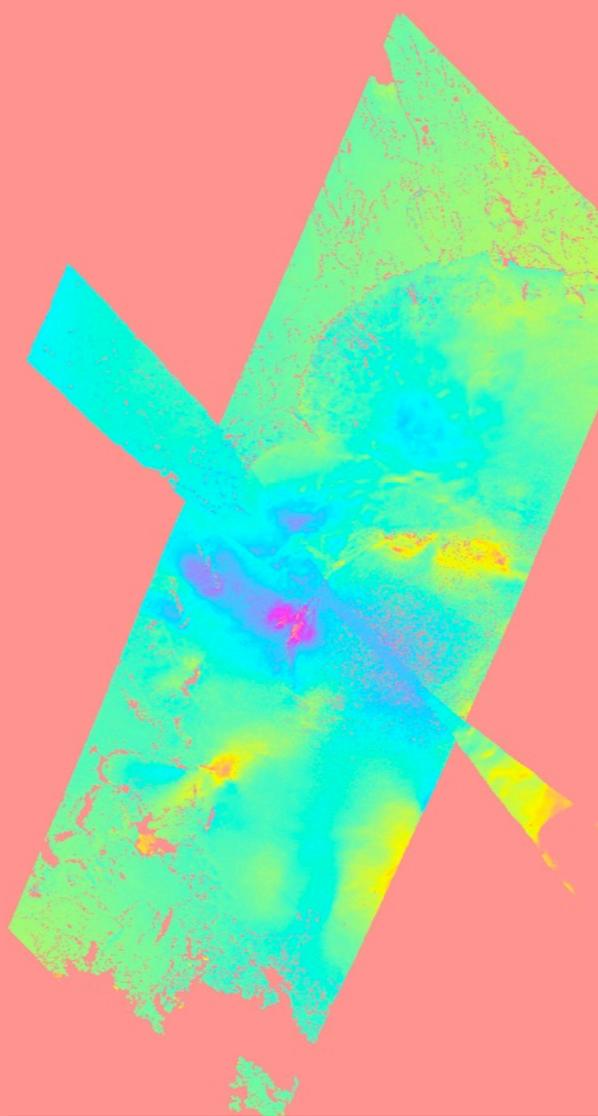


-  Image 1
-  Image 2
-  Image 3
-  Image 4
-  Image 1 & 2
-  Image 1 & 3
-  Image 1 & 4
-  Image 2 & 3
-  Image 2 & 4
-  Image 3 & 4
-  Image 1 & 2 & 3
-  Image 1 & 2 & 4
-  Image 1 & 3 & 4
-  Image 2 & 3 & 4
-  Image 1 & 2 & 3 & 4

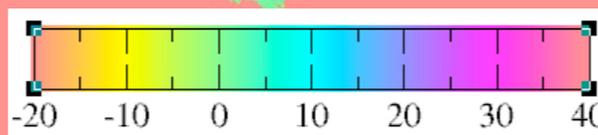


3-D Vector  
Velocity  
Estimates

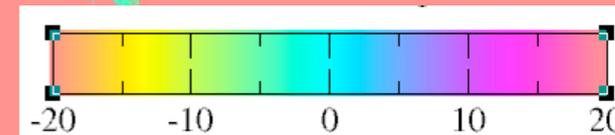
- Sample vector deformation products generated from UAVSAR data collected in May, 2009 over the Hofsjkull glacier.
- Data from headings of  $20^\circ$ ,  $-160^\circ$ ,  $-40^\circ$  and  $140^\circ$  and were combined to generate vector deformation products.

EastNorthUp

Deformation (cm)



Deformation (cm)



Deformation (cm)